

ТЕХНОЛОГІЯ ХАРЧОВОЇ ТА ЛЕГКОЇ ПРОМИСЛОВОСТІ

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DOI <https://doi.org/10.32782/2663-5941/2026.3.2/32>**Kramarenko D.P.**<https://orcid.org/0000-0003-1353-686X>

Simon Kuznets Kharkiv National University of Economics

Hirenko N.I.<https://orcid.org/0000-0001-6854-8257>

State Institution "Luhansk Taras Shevchenko National University"

DEVELOPMENT OF A MATHEMATICAL MODEL OF THE PROCESS OF EVAPORATION OF A LIQUID FOOD MEDIUM UNDER NON-STATIONARY CONDITIONS

The article presents the development of a mathematical model of the unsteady process of evaporation of a liquid food medium from an inert surface under conditions of natural and forced convection. The relevance of the study is due to the critical energy intensity of dehydration processes in the food industry, which account for 12% to 25% of total industrial energy consumption in developed countries, as well as the growing need for the rational utilisation of secondary raw materials, in particular brewer's spent grain (BSG) with an initial moisture content of 75–80%. The physicochemical complexity of liquid food matrices, characterised by non-Newtonian rheological behaviour, a wide range of initial moisture content (80–95%) and a tendency to case hardening, makes it impossible to apply classical convective approaches without prior mathematical justification. The developed model describes the heat balance of the «liquid-inert surface-drying agent» system as a set of interrelated thermodynamic components: convective heat transfer from the drying agent to the liquid surface and metal plate, heat conduction through the tank wall, internal heat transfer between the bottom and the liquid core, and mass transfer during the phase transition. The system of differential heat balance equations covers the entire range of the unsteady-state process – from material heating to reaching the wet thermometer temperature and complete moisture evaporation. The developed mathematical model is an adequate tool for calculating temperature profiles in a liquid and a metal plate, the kinetics of material layer height reduction and mass transfer intensity at different stages of the process. The model is applicable to a wide class of liquid food media with a high initial moisture content: brewer's grains, fruit and vegetable purees, milk concentrates, and protein suspensions. Prospects for further research include verification of the model at a laboratory installation, its extension to describe non-Newtonian systems with variable viscosity, and the development of a digital twin of the drying plant to optimise operating parameters in real time.

Keywords: mathematical model, unsteady evaporation, liquid food medium, heat and mass transfer, natural convection, forced convection, beer foam, heat transfer coefficient, mass transfer coefficient, Nusselt number, Grashof number, molecular diffusion, case hardening, dehydration, food industry.

Formulation of the problem. In the modern food industry, dehydration (drying) processes are among the most energy-intensive, complex, but critically important technological stages. According to fundamental studies of heat and mass transfer, thermodynamic processes of removing moisture from raw materials account for 12% to 25% of total industrial energy consumption in developed countries

[1]. Dehydration of liquid and highly viscous media, such as fruit and vegetable purees, juices with pulp, milk concentrates, protein isolates, extracts, suspensions and fermentation broths, is particularly challenging. These materials are characterised by extremely high initial moisture content, often ranging from 80% to 95% [2], and extremely complex rheological behaviour, most often described by non-



Newtonian models (pseudoplastic, dilatant fluids or Bingham plastics) [3].

The physicochemical nature of such systems poses serious barriers to traditional convective drying methods. Water molecules in the food matrix are in different thermodynamic states: from free moisture in macrocapillaries to physicochemically bound (hydration) moisture on the surface of protein and polysaccharide macromolecules [4]. The removal of free moisture is relatively easy, following the classical laws of evaporation from a free surface. However, as the moisture content decreases, the energy of binding of moisture to the material increases exponentially, requiring significantly higher heat input for the phase transition [5]. In addition, the low thermal conductivity of viscous products leads to uneven heating of the material volume. Rapid evaporation of moisture from the surface layers under conditions of slow internal diffusion causes the formation of a dense, impermeable crust (the so-called «case hardening» phenomenon), which effectively blocks further migration of moisture from the deeper layers. This leads to overheating of the surface, thermal degradation of the product, loss of its biological value and organoleptic properties [6]. All this leads to an urgent and urgent need to move from classical heat treatment methods to innovative dehydration technologies that can intensify mass transfer without destroying the nutritional profile of the product.

Analysis of recent research and publications.

The drying of agricultural waste represents a separate large-scale engineering challenge. Brewer's Spent Grain (BSG) is the main solid by-product of the brewing industry, accounting for about 85% of all waste in the industry. From a chemical point of view, BSG is a complex lignocellulosic matrix (barley grain hulls) containing a significant amount of hemicellulose, lignin, and up to 26% of valuable proteins and lipids [7; 25]. The initial moisture content of fresh pellets reaches 75-80%, which, combined with its rich nutritional composition, makes it an ideal environment for the rapid growth of pathogenic microflora and mould. Without dehydration, the pellet deteriorates within 24 hours, forcing breweries to dispose of it as low-margin feed or simply send it to landfill.

Successful dehydration of brewer's grains transforms this waste into a highly valuable raw material: it is used for thermal energy generation (biofuels) [8], extraction of proteins and enzymes [9], development of innovative edible bioplastic films [10], and as a functional ingredient in the baking industry. However, the complex structure of the

capillary-porous matrix of BSG, where a significant proportion of water is tightly bound, makes drying difficult.

Designing and scaling up modern drying equipment for plastic, liquid and complex products without prior mathematical modelling is an engineering anachronism. The transition from laboratory scale to industrial lines is accompanied by nonlinear changes in hydrodynamics, mass and heat transfer, as a result of which deviations in process parameters by even a few percent can cause product burning, localised overheating zones or the release of underdried products with an increased risk of microbiological spoilage.

Mathematical modelling is an indispensable tool for solving these problems in three key ways. Firstly, it minimises scaling risks by calculating the hydrodynamic and thermal similarity of the devices, which makes it possible to accurately reproduce drying profiles when the volume of the chambers is increased by hundreds of times [11]. Secondly, modern models allow for total optimisation of energy consumption by calculating heat balances in real time, designing heat recovery systems and hybrid heat pump systems, and modelling of solar automatic dryers confirms their payback in less than 2.5 years [12]. Thirdly, controlling the temperature distribution within the material allows you to track the «temperature history» of each product particle, ensuring the preservation of thermolabile compounds such as vitamins in purees or tannins in extracts [13].

Thus, the development of mathematical models of evaporation and drying processes is a scientifically justified necessity and an integral part of the modern approach to the design of food equipment. The use of such models ensures not only the technological reliability and energy efficiency of the designed devices, but also guarantees the quality and safety of the final product, which meets the requirements of current food industry standards and the principles of evidence-based engineering.

Task statement. We set the overall goal of developing an efficient process for drying a liquid food medium, which includes brewer's grains. Therefore, the purpose of the article was to develop a mathematical model of the process of evaporation of a liquid food medium under non-stationary conditions.

Outline of the main material of the study. In order to model the process of drying a liquid food medium with a mixed heat supply, we will develop a mathematical model of the unsteady process of evaporation of liquid solutions from a neutral surface under the condition of natural convection.

During the drying process, liquid solutions go through heating stages as a result of heat transfer from heated air to the material to be dried and from the heated neutral surface on which the material is located. In addition, the system evaporates moisture from the surface of the solution and transfers moisture within the material.

This article is devoted to the development of a mathematical model of the unsteady process of evaporation of liquid solutions from an inert surface under natural and forced convection. A surface made of a material that does not chemically interact with the material to be dried (stainless steel) is taken as neutral. The mathematical description is given on the example of water evaporation, the properties of which are well known [14,15].

A portion of the material (water) is poured into a rectangular metal tank, which is placed in a thermostat. The lower part of the tank is represented as a stainless steel plate (Fig. 1).

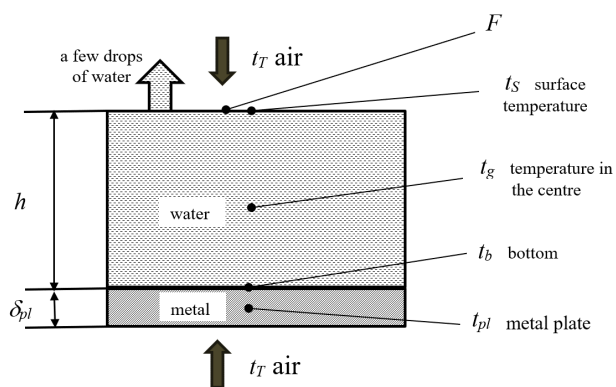


Fig 1. Scheme of heat supply

The surface of the water is in contact with air. From below, the water is bounded by a metal plate, which also exchanges heat with the air. In the process of convective drying, the water temperature rises from the initial t_0 , °C, to the wet thermometer temperature t_v , °C, at which its complete evaporation occurs. The heat transferred from air to water consists of two components: heat transferred by convection to the water surface

$$F\alpha_T(t_T - t_S)d\tau, \quad (1)$$

and the heat transferred by convection from the air to the metal plate

$$F\alpha_{pl}(t_T - t_{pl})d\tau, \quad (2)$$

where F is the heat transfer surface, m^2 ; α_T is the heat transfer coefficient from air to the water surface, $W/(m^2 \cdot K)$; α_{pl} is the heat transfer coefficient from air to the plate (metal), $W/(m^2 \cdot K)$; t_g is the temperature of water in the core, °C; t_b is the temperature of water at the bottom of the bubble, °C; t_{pl} is the temperature of

the metal plate from the side of air, °C; t is the drying time of the material, s; h is the height of the water layer, m; $\delta_{(pl)}$ is the thickness of the plate, m.

Heat is supplied by the non-equimolar transfer of matter [16, 17].

$$G_g t_g dG, \quad (3)$$

Let us assume the temperature of the drying agent (air) in the thermostat t_T , °C, which remains constant throughout the entire process of water evaporation. Relative air humidity φ , %, heat and mass transfer surface F , m^2 , surface temperature of water in contact with air t_S , °C.

In a non-stationary drying process, the supplied heat is consumed: for the evaporation of moisture from water:

$$\left[\frac{F\beta_T(p_{st} - p_p) \times r}{\frac{RT_S}{M_w}} \right] d\tau, \quad (4)$$

for heating the steel plate:

$$d(G_{pl}C_{pl}t_{pl}), \quad (5)$$

and heating the water:

$$d(G_g C_g t_g), \quad (6)$$

where M_w is the molecular weight of water, $kg/kmol$; $p_{(st)}$ is the pressure of saturated water vapour, Pa, at the water surface temperature t_S ; p_p is the partial pressure of water vapour, Pa, in the thermostat; β_T is the mass transfer coefficient from water to air, m/s ; r is the specific heat of vapour of water, J/kg , at the water surface temperature t_S ; G_{pl} is the mass of the plate, kg ; C_{pl} is the specific heat capacity of the plate material, $W/(kg \cdot K)$; G_g is the mass of the water sample, kg ; C_g is the specific heat capacity of water, $W/(kg \cdot K)$; R is the universal gas constant, $R = 8314 J/(kmol \cdot K)$; T_S is the absolute water surface temperature, K ; $dt_{(g)}$ is the increase in water temperature over time $d\tau$; $dt_{(pl)}$ is the increase in plate temperature over time $d\tau$.

Taking into account expressions (1) – (6), the heat balance equation will be as follows:

$$F\alpha_T(t_T - t_S)d\tau + F\alpha_{pl}(t_T - t_{pl})d\tau + C_g t_g dG = \frac{F\beta_T(p_{st} - p_p) \times r}{\frac{RT_S}{M_w}} d\tau + d(G_g C_g t_g) + d(G_{pl} C_{pl} t_{pl}), \quad (7)$$

After opening the brackets, considering C_g and $C_{(pl)}$ to be constant, we obtain:

$$F\alpha_T(t_T - t_S)d\tau + F\alpha_{pl}(t_T - t_{pl})d\tau = \frac{F\beta_T(p_{st} - p_p) \times r}{\frac{RT_S}{M_w}} d\tau + C_g t_g dt_g + G_{pl} C_{pl} dt_{pl}, \quad (8)$$

In the process of evaporation during the time $d\tau$, the mass of water changes by the value dG , kg:

$$dG = - \left[\frac{F\beta_T(p_{st} - p_p)}{\frac{RT_s}{M_w}} \right] d\tau, \quad (9)$$

During the time $\tau + d\tau$ the mass of water will be equal to $G_{\tau+d\tau}$:

$$G_{\tau+d\tau} = G_\tau + dG, \quad (10)$$

and the thickness of the water layer (height) will be equal to $h_{\tau+d\tau}$

$$h_{\tau+d\tau} = h_\tau + dh, \quad (11)$$

where G_τ , h_τ are the mass and thickness of the water layer at time τ , kg; dh is the change in the thickness of the water layer, m.

The term $\alpha_{pl}(t_r - t_{pl})$ in equation (8) describes the amount of heat transferred from air to glass by convection. Part of this heat is used to heat the glass, while the other part is transferred through the glass to the water and is used to heat the water and evaporate it. Heat loss to evaporation of water will occur when the temperature of the water in the core t_g becomes higher than the temperature of the water surface t_s .

Let us consider natural convection. To calculate the coefficient convective heat transfer coefficient α_r for media with $Pr \geq 0.5$, the following formula is recommended when considering convection in an unlimited volume [18,19]:

$$Nu_r = k(Pr \times Gr)^n, \quad (12)$$

where Nu_r is the Nusselt thermal criterion; Pr is the Prandtl criterion; Gr is the Grasthoff criterion; k , n are coefficients depending on the product $Pr \times Gr$.

$$Nu_r = \frac{\alpha_r l}{\lambda}, \quad (13)$$

$$Pr = \frac{\nu}{a}, \quad (14)$$

$$Gr = \frac{g l^3 \beta (t_r - t_s)}{\nu^2}, \quad (15)$$

where g is the acceleration of free fall $g = 9.81 \text{ m/s}^2$; l is the defining linear dimension (characteristic length), m; β is the temperature coefficient of volume expansion of air, K^{-1} ; ν is the kinematic viscosity of air, m^2/s ; a is the thermal conductivity coefficient, m^2/s ; λ is the specific thermal conductivity of air, $\text{W}/(\text{m}\cdot\text{K})$.

The recommended values of k and n in formula (12) are given in Table 1.

It is recommended to increase the value of k for a plate with the heating side facing upwards by 30

%, and to decrease it for a plate with the heating side facing downwards by 30 % compared to the value of k given in Table 1 [18,19].

Table 1

Estimated values of k and n in formula (12)

$Pr \times Gr$	k	n
$< 10^{-3}$	0,45	0
$10^{-3} \div 5 \cdot 10^2$	1,18	1/8
$5 \cdot 10^2 \div 2 \cdot 10^7$	0,54	1/4
$> 2 \cdot 10^7$	0,14	1/3

The mass transfer coefficient β_T , m/s, by analogy with heat transfer, is determined from the Nusselt diffusion criterion Nu_D

$$Nu_D = \frac{\beta_T l}{D}, \quad (16)$$

where D is the molecular diffusion coefficient, m^2/s .

To determine the value of D for the water-air system, we use the formula [20,21]:

$$D = D_0 \left(\frac{T_{abs}}{237} \right)^{1,8} \times \frac{101,3}{p_{bar}}, \quad (17)$$

where T_{abs} is the absolute temperature of the mixture of water vapour and air, K; p_{bar} is the barometric pressure, kPa, at which the process of moisture evaporation occurs.

The diffusion coefficient D_0 at $t = 0 \text{ }^\circ\text{C}$ and barometric pressure $p_{bar} = 101.3 \text{ kPa}$ for the air-water vapour mixture is $D_0 = 0.0784 \text{ m}^2/\text{h}$ or $D_{(0)} = 2.178 \times 10^{-5} \text{ m}^2/\text{s}$ [20,21].

The value of α_{pl} for the case of heat transfer from air to a solid horizontal surface with a constant wall temperature, which can be considered the temperature of the metal plate on the air side t_{pl} during the period of constant drying rate, is determined by expression [22]:

$$Nu = 0.667(Ra\psi(Pr))^{1/5}, \quad (18)$$

where Ra is the Rayleigh criterion, $Ra = Gr \times Pr$; $\psi(Pr)$ is a function of the Prandtl criterion.

$$\psi(Pr) = \left[1 + \left(\frac{0,492}{Pr} \right)^{9/16} \right]^{-16/9}, \quad (19)$$

Equation (19) is recommended for all cases of natural convection in boundary layers [22].

A characteristic geometric dimension in the criterion equations (12) and (18) is the ratio of the area of the heat and mass transfer surface (water surface) to its perimeter:

$$l = \frac{S}{P}, \quad (20)$$

where S is the surface area, m^2 , and P is the perimeter, m.

To determine the surface temperature at which water evaporates, we write the balance equation:

$$F\alpha_T(t_T - t_S)d\tau + F\alpha_{g,2}(t_g - t_S)d\tau = \frac{F\beta_T(p_{st} - p_p) \cdot r}{RT_S} d\tau, \quad (21)$$

where $\alpha_{g,2}$ is the heat transfer coefficient between the surface and the core of water (Fig. 2).

The direction of heat flow in equation (21) will be determined by the driving

heat transfer force $t_g - t_S$. In the case of $t_g < t_S$, the heat flux will be negative, i.e., the supplied heat will be spent on heating the water and its evaporation.

From equation (21) we obtain:

$$t_S = \frac{\alpha_T t_T + \alpha_{g,2} t_g - [\beta_T(p_{st} - p_p) \cdot r] \cdot M_w / RT_S}{\alpha_T + \alpha_{g,2}}, \quad (22)$$

It is known [23] that the temperature difference between the surface of a body and the surrounding fluid leads to a density gradient that causes the fluid to move.

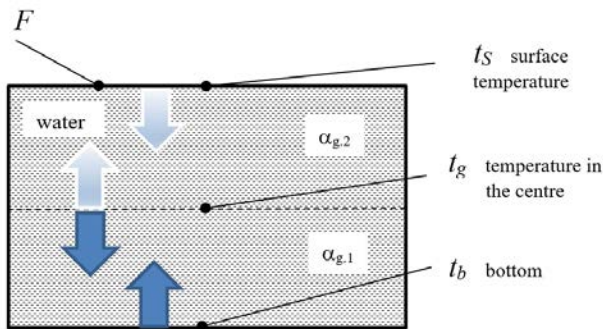


Fig. 2. Scheme of heat transfer between the surface and the core

The intensity of heat transfer between the body and the fluid increases compared to pure heat conduction. This movement of the fluid associated with the transfer of heat leads to free convection. In our case, the body is a metal plate to which heat is supplied from the air, and the surrounding fluid is water.

To determine the temperatures of t_b and t_{pl} , write down the heat balance (Fig. 2). The direction of heat flows will be determined by the corresponding temperature difference $(t_S - t_g)$ and $(t_b - t_g)$.

$$\alpha_{g,1}(t_b - t_g)Fd\tau = GC_g dt_{g,1}, \quad (23)$$

$$\alpha_{g,2}(t_S - t_g)Fd\tau = GC_g dt_{g,2}, \quad (24)$$

The left side of equations (23) and (24) describes the heat transfer from the boundary (bottom, surface) to the water core. The right-hand side shows the amount of heating or cooling of the liquid depending on the direction of heat flow. Adding the left and right

sides of equations (23) and (24), we obtain

$$\alpha_{g,1}(t_b - t_g)Fd\tau + \alpha_{g,2}(t_S - t_g)Fd\tau = GC_g(dt_{g,1} + dt_{g,2}) = GC_g dt_g, \quad (25)$$

From equation (25), we determine: t_b

$$t_b = \frac{GC_g dt_g - \alpha_{g,2}(t_S - t_g)Fd\tau + \alpha_{g,1} t_g Fd\tau}{\alpha_{g,1} Fd\tau}, \quad (26)$$

The heat that has passed through the metal plate due to thermal conductivity is equal to the heat removed from the plate to the water due to heat transfer:

$$\frac{\lambda_{pl}}{\delta_{pl}}(t_{pl} - t_b) = \alpha_{g,1}(t_b - t_g), \quad (27)$$

From equation (27), we find the temperature of the metal plate on the air side: t_{pl}

$$t_{pl} = t_b + \alpha_{g,1}(t_b - t_g) \frac{\delta_{pl}}{\lambda_{pl}}, \quad (28)$$

The calculation formulas for determining the heat transfer coefficients for free convection in the liquid $\alpha_{g,1}$ and $\alpha_{g,2}$ are as follows [24]:

at $10^{-3} < Pr \times Gr < 500$

$$\alpha = A_1 \left(\frac{t}{l} \right)^{1/8}, \quad (29)$$

at $500 < Pr \times Gr < 2000000$

$$\alpha = A_2 \left(\frac{t}{l} \right)^{1/4}, \quad (30)$$

at $Pr \times Gr > 2000000$

$$\alpha = A_3 \left(\frac{t}{l} \right)^{1/3}, \quad (31)$$

The values of the coefficients A for water are given in Table 2 [24].

Table 2
Values of the coefficients A_1, A_2, A_3 in formulas (29)-(31)

°C	0	20	40	60	80	100	150	200
A_1	8,0	11,3	13,5	15,1	16,3	17,2	18,7	19,4
A_2	60	96	128	153	176	195	235	262
A_3	88	170	250	312	366	414	522	614

In formulas (29)-(31), we take half the thickness of the water layer $h/2$ as the defining linear dimension.

The above system of equations is a mathematical description of the unsteady evaporation of liquid solutions.

Conclusions. A mathematical model of the unsteady evaporation of a liquid food medium from an inert surface under conditions of natural and forced convection has been developed. The model describes the heat balance of the system as a set of interconnected components: convective heat transfer

from the drying agent to the surface of the liquid and the metal plate, heat conduction through the tank wall and internal heat transfer between the bottom and the core of the liquid. The proposed system of equations covers all stages of the unsteady-state process, from heating the material to reaching the wet bulb temperature and complete moisture evaporation.

To determine the convective heat transfer coefficients, the criterion equations with the Nusselt, Grashof, and Prandtl criteria adapted to the geometry of the system under study were used. The mass transfer coefficient is calculated on the basis of the analogy between heat and mass transfer using the Nusselt diffusion criterion.

The coefficient of molecular diffusion in the water-air system is determined by temperature and barometric dependencies with the reference value $D_0 = 2.178 \times 10^{-5} \text{ m}^2/\text{s}$ at 0°C and a pressure of 101.3 kPa.

The resulting model is an adequate mathematical tool for calculating temperature profiles in the liquid and the plate, the kinetics of the material layer height reduction and the mass transfer rate at different stages of evaporation. The model is applicable to

liquid food media with a significant initial moisture content, including brewer's grains with a moisture content of 75-80%, fruit and vegetable purees, milk concentrates, and protein suspensions.

A promising direction is the verification of the developed mathematical model by comparing the calculated data with the results of a physical experiment in a laboratory installation with controlled parameters of the drying agent. It is important to extend the model to describe the behaviour of non-Newtonian liquids – pseudoplastic and dilute systems – taking into account the variable viscosity during the concentration process. An important task is to study the phenomenon of «case hardening» and develop a criterion for its occurrence based on the obtained temperature profiles. Further study is required to validate the model on specific food matrices, such as brewer's grains, fruit purees and protein isolates, with the determination of correction factors for each product class. It is also promising to develop a digital twin of the drying plant based on the proposed model for real-time optimisation of operating parameters and design of heat recovery systems.

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Крамаренко Д.П., Гіренко Н.І. РОЗРОБКА МАТЕМАТИЧНОЇ МОДЕЛІ ПРОЦЕСУ ВИПАРОВУВАННЯ РІДКОГО ХАРЧОВОГО СЕРЕДОВИЩА ПРИ НЕСТАЦІОНАРНИХ УМОВАХ

У статті представлено розробку математичної моделі нестационарного процесу випаровування рідкого харчового середовища з інертної поверхні за умов природної та вимушеної конвекції. Актуальність дослідження зумовлена критичною енергосмістю процесів зневоднення у харчовій промисловості, на частку яких припадає від 12% до 25% загального промислового енергоспоживання в розвинених країнах, а також зростаючою потребою в раціональній утилізації вторинної сировини – зокрема пивної дробини (Brewer's Spent Grain, BSG) з початковим вологовмістом 75–80%. Фізико-хімічна складність рідких харчових матриць, що характеризуються неньютонівською реологічною поведінкою, широким діапазоном початкової вологості (80–95%) та схильністю до явища «case hardening», унеможливає застосування класичних конвективних підходів без попереднього математичного обґрунтування. Розроблена модель описує тепловий баланс системи «рідина–інертна поверхня–сушильний агент» як сукупність взаємоз'язаних термодинамічних складових: конвективного теплопідводу від сушильного агенту до поверхні рідини та металевій пластини, теплопровідності через стінку резервуару, внутрішньої теплоїдачі між дном і ядром рідини, а також масоперенесення при фазовому переході. Система диференціальних рівнянь теплового балансу охоплює весь діапазон нестационарного процесу – від прогрівання матеріалу до досягнення температури мокрого термометра та повного випаровування вологи. Розроблена математична модель є адекватним інструментом для розрахунку температурних профілів у рідині та металевій пластині, кінетики зменшення висоти шару матеріалу та інтенсивності масовідачі на різних етапах процесу. Модель придатна для застосування до широкого класу рідких харчових середовищ із значним початковим вологовмістом: пивної дробини, фруктових і овочевих пюре, молочних концентратів та білкових суспензій. Перспективами подальших досліджень є верифікація моделі на лабораторній установці, її розширення для опису неньютонівських систем зі змінною в'язкістю, а також розробка цифрового двійника сушильної установки для оптимізації режимних параметрів у реальному часі.

Ключові слова: математична модель, нестационарне випаровування, рідке харчове середовище, тепломасообмін, природна конвекція, вимушена конвекція, пивна дробина, коефіцієнт теплоїдачі, коефіцієнт масовідачі, критерій Нуссельта, критерій Грасгофа, молекулярна дифузія, «case hardening», зневоднення, харчова промисловість.

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